MTH 150 Precalculus Chapter 4 Problems

Roxy Hepburn

November 2, 2020

1 Chapter 4

1.1 P.263, 7-12, (Pick 1) ; P.264, 13-22, (Pick 1) ; P.276, 23-28, (Pick 2)

pg.263, #7: A population numbers 11,000 organisms initially and grows by 8.5% each year. Write an exponential model for the population.

$$P(t) = 11,000(1.085)^t$$

I did not struggle with this problem. The formula for an exponential model is $f(x) = ab^x$. A is the given initial amount, 11,000 organisms, and b is the percent in which the rate increases plus 1, 1.085. Plug those values into the formula, and replace f(x) with P(t) to reflect the information in the problem.

pg.264, #13: Find a formula for an exponential function passing through the two points (0, 6) ; (3, 750).

$$f(x) = ab^x, a = 6$$
$$f(x) = 6b^x, 750 = 6b^3, 125 = b^3, b = 5$$
$$f(x) = (6)(5)^x$$

I did not struggle with this problem. When 2 points are given and the function needs to be found, if one of the points is in (0, x) format, then the x value will be equal to a. In this problem's case, the point is (0, 6), and so a = 6. Then, the other point was plugged back into the formula and solved for b, which is b = 5. After finding the values, for the variables, they were plugged into the exponential model.

pg.276, #23: Describe the long run behavior of the function, $f(x) = -5(4^x) - 1$ as $x \to \infty$ and $x \to \infty$.

As $x \to \infty, f(x) \to -\infty$
As $x \to -\infty, f(x) = -1$

I find long run behavior problems easy when looking at the function graphed. The shape of this function graphed comes from the left side, positive and crosses into the right side as it exponentially curves down dramatically into the negative as the line moves further right.

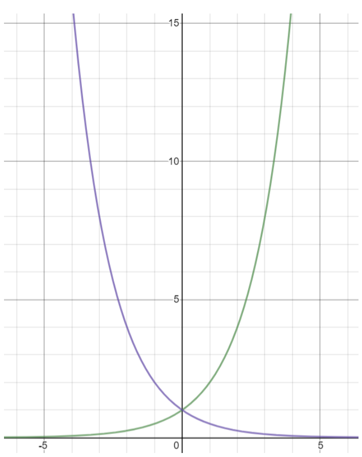
pg.276, #25: Describe the long run behavior of the function, $f(x) = 3(\frac{1}{2})^x - 2$ as $x \to \infty$ and $x \to \infty$.

As $x \to \infty, f(x) \to -2$
As $x \to -\infty, f(x) \to \infty$

I found this long run behavior problem very easy. When graphed, the line comes from the positive (top), left side decreasing exponentially, and once it passes into the right side negative, the line drastically curves and basically flattens as it approaches the right side further.

1.2 P.275, 11-16, (Pick 1) ; P.275, 17-22, (Pick 1) ; P.264-266, 23-27, (Pick 1)

pg.275, #11: Sketch a graph of the following transformation of $f(x) = 2^x$ to $f(x) = 2^{-x}$.



I found the transformation problem very easy. All I had to do was plug both of the given equations into desmos and screen capture the image.

pg.275, #17: Starting with the graph of $f(x) = 4^x$, find a formula for the function that results from shifting f(x) 4 units upwards.

$$f(x) = 4^x + 4$$

I find writing formulas for transformations problems pretty easy. To shift a function upward, you take the entire original function and add the shift in units to it, so it goes from $f(x) = 4^x$ to $f(x) = 4^x + 4$.

pg.264, #23: A radioactive substance decays exponentially. A scientist begins with 100 milligrams of a radioactive substance. After 35 hours, 50 mg of the substance remains. How many milligrams will remain after 54 hours?

$$f(x) = ab^x, f(x) = 100b^x$$

$$50 = 100b^{35}, 0.5 = b^{35}, b = 0.98039$$

$$f(x) = 100(0.98039)^x$$

$$f(54) = 100(0.98039)^{54}, f(54) = 100(0.34321), f(54)34.321$$

34.32 milligrams

I found this exponential model word problem not too difficult. To solve it I used the formula $f(x) = ab^x$. 100mg was the initial given amount, and so a = 100. 35 hours and 50mg were plugged into the formula $50 = 100b^{35}$ and solved for b, which b = 0.98039. I then found the function for the problem, $f(x) = 100(0.98039)^x$ and then plugged in the 54 hours that elapsed, $f(54) = 100(0.98039)^{54}$ and solved for f(54) = 34.32mg.

1.3 P.287, 1-8, (Pick 1) ; P.287, 9-16, (Pick 1) ; P.287, 17-24, (Pick 1) ; P.287, 41-56, (Pick 2) ; P.288, 65-72, (Pick 1)

pg.287, #1: Rewrite $log_4(q) = m$ in exponential form.

$$4^m = q$$

I found rewriting logarithmic functions in exponential form easy. To rewrite logarithmic functions in the form $log_x(y) = z$, they take the form of $x^z = y$ in exponential form.

pg.287, #9: Rewrite $4^x = y$ in logarithmic form.

$$log_4(y) = x$$

I found rewriting exponential functions into logarithmic functions simple. It is the reverse of rewriting logarithmic functions, and so they go from the form of $x^z = y$ into the form of $log_x(y) = z$.

pg.287, #17: Solve for x in the function $log_3(x) = 2$.

$$3^2 = x, x = 9$$

I found solving for x in logarithmic functions easy. First I converted the function into exponential form from $log_3(x) = 2$ to $3^2 = x, x = 9$, and then solved for x, which is x = 9.

pg.287, #41: Solve for the variable in the equation $5^x = 14$.

$$log5^x = log14$$

$$xlog5 = log14$$

$$x(0.69897) = 1.14613, x = 1.6397$$

I found solving for the variable in this equation very simple. To do it, first I converted the equation into logarithmic form. I then used logarithmic exponent properties that pull out the exponent and put it in front of the equation to from $log5^x = log14$ to $xlog5 = log14$ and then solved for x, which is x = 1.6397.

pg.287, #43: Solve for the variable in the equation $7^x = \frac{1}{15}$.

$$log7^x = log\frac{1}{15}$$

$$xlog7 = log\frac{1}{15}$$

$$x(0.8451) = -1.1761, x = -1.3917$$

I found finding the variable of this equation very easy. I converted the equation from exponential into logarithmic form and used its exponent properties to pull out in front the exponent to $xlog7 = log\frac{1}{15}$. I then simplified and solved for x, which is x = -1.3917 pg.288, #65: The population of Kenya was 39.8 million in 2009 and has been growing by about 2.6% each year. If this trend continues, when will the population exceed 45 million?

$$P(t) = 39,800,000(1.026)^t$$
$$45,000,000 = 39,800,000(1.026)^t, 1.13065 = (1.026)^t$$
$$xlog(1.026) = log(1.13065), x(0.01115) = 0.05333, x = 4.784$$

4.784 years

I found this word problem not very difficult. I used the formula $f(x) = ab^x$ to make and solve the equation for t when P(t) is used in the function. The created equation was $P(t) = 39,800,000(1.026)^t$. 45,000,000 was imputed as P(t) and once solved, t = 4.784.

1.4 P.298, 1-16, (Pick 1) ; P.298, 17-26, (Pick 1) ; P.299, 27-48, (Pick 1)

pg.298, #1: Simplify the logarithmic expression, $log_3(28) - log_3(7)$, using logarithm properties.

$$log_3(28/7) = log_3(4)$$

Simplifying logarithmic expressions using logarithmic properties is very easy. The logarithmic property of subtraction is that when in $log_x(y) - log_x(z)$ it can also be seen as $log_x(y/z)$. And so $log_3(28) - log_3(7)$ gets rewritten as $log_3(28/7)$ and then simplified to $log_3(4)$.

pg.298, #17: Use logarithm properties to expand $log(\frac{x^{15}y^{13}}{z^{19}})$

$$log(x)^{15} + log(y)^{13} - log(z)^{19}$$
$$15log(x) + 13log(y) - 19log(z)$$

Expanding logarithmic equations using logarithmic properties is very easy. The logarithmic property of subtraction is that when in $log_x(y) - log_x(z)$ it can also be seen as $log_x(y/z)$. The logarithmic property of addition is that when in $log_x(y) + log_x(z)$ it can also be seen as $log_x(yz)$. So, for $log(\frac{x^{15}y^{13}}{z^{19}})$, to expand $(x^{15}y^{13})$ it becomes $15log(x) + 13log(y)$, and to divide it by $19log(z)$ subtract it, to become $15log(x) + 13log(y) - 19log(z)$.

pg.299, #27: Solve for the variable in $4^{4x-7} = 3^{9x-6}$.

$$(4x - 7)(log4) = (9x - 6)(log3)$$

$$(4x - 7)(0.602) = (9x - 6)(0.477)$$

$$2.408x - 4.214 = 4.293x - 2.862$$

$$-1.885x = 1.352, x = -0.717$$

I found solving for the variable in the equation not too difficult. To solve for the variable, I put the exponential form equation into logarithmic form, $(4x - 7)(log4) = (9x - 6)(log3)$, and then solve for x, which is x = x = -0.717.

1.5 P.306, 1-16, (Pick 4)

pg.306, #1: Find the domain and vertical asymptote of $f(x) = log(x - 5)$.

$$y = log_{10}(x - 5)$$

$$10^y = x - 5$$

$$x - 5 = 0, x = 5$$

Vertical Asymptote: x = 5

$$x - 5 > 0, x > 5$$

Domain: x >5

Finding the domain and vertical asymptote of an equation is easy. To do this, I converted the equation into exponential form and then solved for x when it was equal to and greater than the answer, as in x = 5 and x >5.

pg.306, #3: Find the domain and vertical asymptote of $f(x) = ln(3 - x)$.

$$y = log_e(3 - x)$$

$$e^y = 3 - x$$

$$3 - x = 0, x = 3$$

Vertical Asymptote: x = 3

$$3 - x > 0, x > 3$$

Domain: x >3

It was simple to find the domain and vertical asymptote of the equation. I converted the equation into exponential form, $e^y = 3 - x$, and then solved for x when it was equal to and greater than the answer, as in x = 3 and x >3.

pg.306, #5: Find the domain and vertical asymptote of $f(x) = log(3x + 1)$.

$$y = log_{10}(3x + 1)$$

$$10^y = 3x + 1$$

$$3x + 1 = 0, 3x = -1, x = -1/3$$

Vertical Asymptote: x = -1/3

$$3x + 1 > 0, 3x > -1, x > -1/3$$

Domain: x >-1/3

I did not struggle with finding the domain and vertical asymptote of the equation. I converted the equation into exponential form, $10^y = 3x + 1$ and then solved for x when it was equal to and greater than the answer, as in x = -1/3 and x >-1/3.

pg.306, #7: Find the domain and vertical asymptote of $f(x) = log(2x + 5)$.

$$y = log_{10}(2x + 5)$$
$$10^y = 2x + 5$$
$$2x + 5 = 0, 2x = -5, x = -5/2$$

Vertical Asymptote: x = -5/2

$$2x + 5 > 0, 2x > -5, x > -5/2$$

Domain: x >-5/2

I found it easy to find the domain and vertical asymptote of the equation. First, I rewrote the equation into exponential form, $10^y = 2x + 5$, and then solved for x when it is equal to or greater than the answer, as in x = -5/2 and x >-5/2.

1.6 P.322-324, 1-22, (Pick 4) ; P.325-327, 29-39, (Pick 4)

pg.322, #1: You go to the doctor and he injects you with 13 milligrams of radioactive dye. After 12 minutes, 4.75 milligrams of dye remain in your system. To leave the doctor's office, you must pass through a radiation detector without sounding the alarm. If the detector will sound the alarm whenever more than 2 milligrams of the dye are in your system, how long will your visit to the doctor take, assuming you were given the dye as soon as you arrived and the amount of dye decays exponentially?

$$m(t) = ab^t, a = 13, m(t) = 13b^t$$
$$4.75 = 13b^12, 0.3654 = b^{12}, b = 0.3654^{1/12}, b = 0.9195$$
$$m(t) = (13)(0.9195)^t, 2 = (13)(0.9195)^t, 0.1538 = 0.9195^t$$
$$log(0.1538) = log(0.9195)^t, log(0.1538) = tlog(0.9195)$$
$$-0.8129 = (-0.0364)t, t = 22.31$$

22.31 minutes

I did not struggle with this word problem. I used the formula $f(x) = ab^x$ to solve the problem. A = 13 as the initial amount is 13mg. I then solved for b using (12, 4.75), which is b = 0.9195. I then plugged a and b into the formula to get the equation $m(t) = (13)(0.9195)^t$. I then plugged 2 in for m(t) and solved for t, which is t = 22.31 minutes.

pg.322, #3: The half-life of Radium-226 is 1590 years. If a sample initially contains 200 mg, how many milligrams will remain after 1000 years?

$$h(t) = ab^t$$

$$0.5a = ab^{1590}, 0.5 = b^{1590}, 0.5 = b^{1/1590}, b = 0.99956$$

$$a = 200, h(t) = (200)(0.99956)^t$$

$$h(1000) = (200)(0.99956)^{1000}, h(1000) = (200)(0.646655)$$

$$h(1000) = 129.331$$

129.331 milligrams Radium-226

I did not struggle with this word problem. I used the formula $f(x) = ab^x$ to solve the problem. I solved for b using $0.5a = ab^{1590}$ where the a's on each side of the = cancel out, and b = 0.99956. A = 200 as 200mg is the given initial amount. I then plugged a and b into the formula as $h(t) = (200)(0.99956)^t$. I then solved for h(1000), $h(1000) = (200)(0.99956)^{1000}$ which is h(1000) = 129.331mg.

pg.322, #5: The half-life of Erbium-165 is 10.4 hours. After 24 hours a sample still contains 2mg. What was the initial mass of the sample, and how much will remain after another 3 days?

$$h(t) = ab^t$$

$$0.5a = ab^{10.4}, 0.5 = b^{10.4}, 0.5 = b^{1/10.4}, b = 0.9355$$

$$h(t) = a(0.9355)^t, 2 = a(0.9355)^2 4, 2 = a(0.20198), a = 9.9018$$

$$h(96) = (9.9018)(0.9355)^{96}, h(96) = (9, 9018)(0.00166)$$

$$h(96) = 0.01648$$

0.01648 milligrams Erbium-165

I found this word problem easy. I used the formula $f(x) = ab^x$ to solve the problem. I solved for b using $0.5a = ab^{10.4}$ where the a's cancel out on each side of the =, where b = 0.9355. I then solved for a by plugging in b and (24, 2) and solving as $2 = a(0.9355)^2 4$ to get a = 9.9018. I then plugged a and b into the formula to get $h(t) = (9.9018)(0.9355)^t$. I then solved for h(96) as $h(96) = (9.9018)(0.9355)^{96}$, which is h(96) = 0.01648.

pg.322, #7: A scientist begins with 250 grams of a radioactive substance. After 225 minutes, the sample has decayed to 32 grams. Find the half-life of this substance.

$$32 = 250b^{225}$$

$$8000 = b^{225}, b = 1.0408$$

Half-life: 4.08%

I found this problem to be not too difficult. I used the formula $f(x) = ab^x$ to solve the problem. The only variable not given in the problem is b as in, $32 = 250b^{225}$, and to find the half-life is to find b. I solved for b, which is b = 1.0408. Subtract 1 from 1.0408 (0.0408) and multiply by 100 to get the percentage, 4.08%.

pg.325, #29: The 1906 San Francisco earthquake had a magnitude of 7.9 on the MMS scale. Later there was an earthquake with magnitude 4.7 that caused only minor damage. How many times more intense was the San Francisco earthquake than the second one?

I have struggled significantly with this word problem. I am not sure how to put the magnitudes into logarithmic form, and the solution manual is not help other than tell me to do so, it is very unclear for this problem.

pg.325, #31: One earthquake has magnitude 3.9 on the MMS scale. If a second earthquake has 750 times as much energy as the first, find the magnitude of the second quake.

I also struggled significantly with this problem. The solution manual is unclear on how to start going about this problem. It implies an equation which I am not sure what it is referencing which is leaving me further confused.

pg.325, #33: A colony of yeast cells is estimated to contain 106 cells at time t = 0. After collecting experimental data in the lab, you decide that the total population of cells at time t hours is given by the function $f(t) = 10^6 e^{0.495105t}$. [UW]
a - How many cells are present after one hour?
b - How long does it take of the population to double?
c - Cherie, another member of your lab, looks at your notebook and says: "That formula is wrong, my calculations predict the formula for the number of yeast cells is given by the function. $f(t) = 10^6(2.042727)^{0.693147t}$." Should you be worried by Cherie's remark?
d - Anja, a third member of your lab working with the same yeast cells, took these two measurements: 6 7.246 10 × cells after 4 hours; 6 16.504 10 × cells after 6 hours. Should you be worried by Anja's results? If Anja's measurements are correct, does your model over estimate or under estimate the number of yeast cells at time t?

a - $f(1) = 10^6 e^{0.495105(1)}, f(1) = 1640670.5$

1,640,670.5 bacteria

b - $2,000,000 = 10^6 e^{0.495105t}, 2 = e^{0.495105t}, t = 1.4$

1.4 hours

c - No, when the value of time is small, the quantity is close enough.

d - $f(4) = 10^6 e^{0.495105(4)}, f(4) = 1980420, 1980420 \neq 7.246X10^6$

With the given equation, when 4 hours go by the amount of bacteria is much less than what the classmate found. Yes I should be worried, her model overestimates the number of cells.

I found this problem not too difficult. For a, I plugged 1 into the given formula as $f(1) = 10^6 e^{0.495105(1)}$ and found f(1) = 1640670.5. For b, I doubled the initial amount and used that value, 2,000,000 as f(x), which is $2,000,000 = 10^6 e^{0.495105t}$, and solved for t, which is t = 1.4 hours. For c, the classmate's given equation has a value of time that is small, which means the calculated quantity would be close enough. For d, I plugged in 4 as f(4) and solved as $f(4) = 10^6 e^{0.495105(4)}$, which is f(4) = 1980420. The calculated amount of cells is much less than the classmate's calculated amount, which means they severely overestimated.

pg.327, #37: A cancer cell lacks normal biological growth regulation and can divide continuously. Suppose a single mouse skin cell is cancerous and its mitotic cell cycle (the time for the cell to divide once) is 20 hours. The number of cells at time t grows according to an exponential model. [UW]
a - Find a formula C(t) for the number of cancerous skin cells after t hours.
b - Assume a typical mouse skin cell is spherical of radius $50X10^{-4}cm$. Find the combined volume of all cancerous skin cells after t hours. When will the volume of cancerous cells be $1cm^3$?

I found this word problem extremely difficult. The solution manual is unclear and I was left confused as to where to start for this problem.

1.7 P.335, 9-16, (Pick 4)

pg.335, #9: Use regression to find an exponential function that best fits the data given.

$$y = 776.68(1.4260)^x$$

I did not struggle with using regression to find the exponential function for the given table. I plugged the table into my calculator under the Lists under STAT. I then used STAT then CALC to calculate the variables within the exponential function, which came out to $y = 776.68(1.4260)^x$.

pg.335, #11: Use regression to find an exponential function that best fits the data given.

$$y = 731.92(0.7385)^x$$

Finding the exponential function using regression was very easy. I plugged the table into my calculator under the Lists, and the used STAT then CALC to find the variables of the exponential function, which is $y = 731.92(0.7385)^x$.

pg.335, #13: Total expenditures (in billions of dollars) in the US for nursing home care are shown below. Use regression to find an exponential function that models the data. What does the model predict expenditures will be in 2015?

$$y = 54.96(1.054)^x$$

$$y = 54.96(1.054)^{25}, y = 204.67$$

204.67 billion in expenditures

I did not struggle with this word problem. I plugged the given table into my calculate and found the exponential function to be $y = 54.96(1.054)^x$ using regression. I then plugged in 25 (25 years after initial year) as x, $y = 54.96(1.054)^{25}$ and solved the equation for y to find the expenditures to be 204.67 billion.

pg.335, #15: The average price of electricity (in cents per kilowatt hour) from 1990 through 2008 is given below. Determine if a linear or exponential model better fits the data, and use the better model to predict the price of electricity in 2014.

Exponential Model

$$y = 7.599(1.0161)^x$$

$$y = 7.599(1.0161)^{24}, y = 7.599(1.46715), y = 11.149$$

11.149 cents per kilowatt hour

I did not struggle with this word problem. When looking at the plotted data, it can be seen that an exponential model better fits the data. By using regression and the data in my calculator, the equation, $y = 7.599(1.0161)^x$ was created, and 24 years was used as x, $y = 7.599(1.0161)^{24}$, to get 11.149 cents per kilowatt hour.