MTH 150 Precalculus Chapter 6 Problems

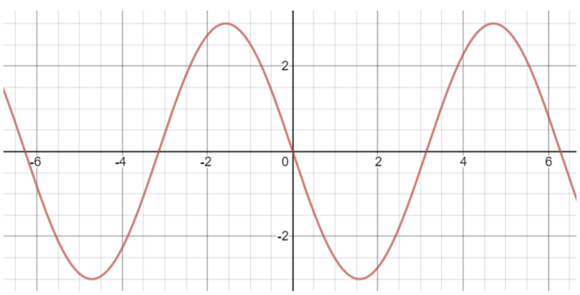
Roxy Hepburn

November 30, 2020

1 Chapter 6

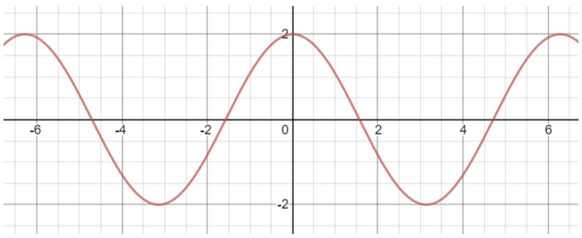
1.1 P.409, 1-4, (Pick 2) ; P.410, 11-16, (Pick 2) ; P.411, 21-24, (Pick 2)

pg.409, #1: Sketch a graph of $f(x) = -3sin(x)$.



I did not struggle with graphing sine functions. I know that when graphed, a sine function will appear wavy. I plugged the equation into desmos and the graph generated matched this trend.

pg.409, #3: Sketch a graph of $f(x) = 2cos(x)$.



I found graphing cosine functions easy as well. Cosine functions will also take on the wavy appearance. I plugged the given equation, $f(x) = 2cos(x)$, into desmos to sketch the it.

pg.410, #11: Find the amplitude, period, horizontal shift, and midline of $y = 3sin(8(x + 4))5$.

Amplitude - 3
Period - $\frac{\pi}{4}$
Horizontal Shift - 4 to the left
Midline - $y = 5$

I did not struggle with finding he amplitude, period, horizontal shift, and midline of $y = 3sin(8(x + 4))5$. I used the base formula, $f(x) = asin(bx + c) + d$. The amplitude is the a value, the coefficient to the sin function, 3. To find the period, I multiplied the absolute value of b, by 2π to get $\frac{\pi}{4}$. The horizontal shift is the bx + c value, which is (x + 4), a positive value means a shift to the left. The midline is the d value of the function, 5.

pg.410, #13: Find the amplitude, period, horizontal shift, and midline of $y = 2sin(3x - 21) + 4$.

Amplitude - 2
Period - $\frac{2\pi}{3}$
Horizontal Shift - 7 to the right
Midline - $y = 4$

I did not struggle with this problem as well. To complete it, I used the base formula $f(x) = asin(bx + c) + d$. The amplitude is the a value, 2. The period is the absolute value of b, 3, multiplied by 2π to get $\frac{2\pi}{3}$. The horizontal shift is the bx + c value, where $3x - 21$ needed to be converted to $3(x - 7)$ to be in the correct format. A negative value means a shift to the right 7 units. The midline is the d value, 4, of the given function.

pg.411, #21: Outside temperature over the course of a day can be modeled as a sinusoidal function. Suppose you know the temperature is 50 degrees at midnight and the high and low temperature during the day are 57 and 43 degrees, respectively. Assuming t is the number of hours since midnight, find a function for the temperature, D, in terms of t.

Amplitude - 7
Midline - $y = 50$

$$D(t) = -7sin(\frac{2\pi}{24}t) + 50$$

$$D(t) = -7sin(\frac{\pi}{12}t) + 50$$

I found the problem semi difficult and so I used the solution manual to guide me into how to complete the math in the problem. I knew to find the formula it would be in the format $f(x) = asin(bx+c)+d$. To start I found the midline by finding the halfway point between the maximum and minimum temperatures, 50. Next, I found the amplitude, which is the difference between the max/min(s). I knew to make the sine negative so that the temperatures were at the correct times. I then plugged the values in and simplified to get the equation $D(t) = -7sin(\frac{\pi}{12}t) + 50$.

pg.411, #23: A Ferris wheel is 25 meters in diameter and boarded from a platform that is 1 meters above the ground. The six o'clock position on the Ferris wheel is level with the loading platform. The wheel completes 1 full revolution in 10 minutes. The function h(t) gives your height in meters above the ground t minutes after the wheel begins to turn.

A: Find the amplitude, midline, and period of h(t).
B: Find a formula for the height function h(t).
C: How high are you off the ground after 5 minutes?

a: Amplitude - 12.5 meters
Midline - 13.5 meters
Period - 10 minutes
b: $h(t) = -12.5cos(\frac{2\pi}{10}t) + 13.5$
c: 26 meters

$$h(5) = -12.5cos(\frac{2\pi}{10}(5)) + 13.5$$

$$h(5) = -12.5cos(\pi) + 13.5$$

$$h(5) = 12.5 + 13.5$$

$$h(5) = 26$$

I struggled with this word problem and so I used the solution manual to understand what to do and guide me through the math. To find the amplitude, I took the maximum and minimum heights of the Ferris wheel and found the half distance between them, 12.5 meters. The midline is the center of the Ferris wheel, which is 1 extra meter more than the amplitude. The period is 10 minutes as that is th amount of time 1 rotation period takes. I then plugged those variables into the base formula $f(x) = asin(bx + c) + d$. I replaced sine with cosine and made it negative to match the data properly. To solve how high off the ground you would be after 5 minutes, I plugged 5 in for t as $h(5) = -12.5cos(\frac{2\pi}{10}(5)) + 13.5$ and solved to get 26 meters.

1.2 P.419, 5-10, (Pick 2) ; P.420, 15-16, (Pick 1) ; P.421, 21-26, (Pick 2) ; P.421, 27-28, (Pick 1)

pg.419, #5: Find the period and horizontal shift of $f(x) = 2tan(4x - 32)$.

Period - $\frac{\pi}{4}$
Horizontal Shift - 8 units to the right

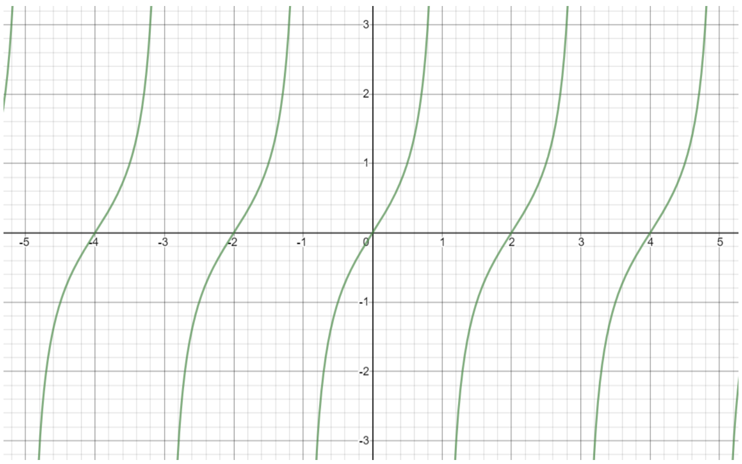
I did not struggle with finding the period and horizontal shift of the given function. I used the base formula $f(x) = asin(bx + c) + d$ where the period is the absolute value of b, 4, multiplied by 2π, $\frac{\pi}{4}$, and the horizontal shift is bx + c where positive is left and negative is right. To find the horizontal shift $(4x - 32)$ needed to be in the correct form, $4(x - 8)$. After that the horizontal shift is visible as negative 8 or 8 units to the right.

pg.419, #7: Find the period and horizontal shift of $h(x) = 2sec(\frac{\pi}{4}(x+1))$.

Period - 8
Horizontal Shift - 1 unit to the left

I found this type of problem very easy. To complete it, I used the base formula $f(x) = asin(bx + c) + d$. The period is the absolute value of b, $\frac{\pi}{4}$ multiplied by 2π which simplifies to 8. The horizontal shift is bx + c where positive is left and negative is right and so positive 1 is 1 unit to the left.

pg.420, #15: Sketch a graph of $j(x) = tan(\frac{\pi}{2}x)$.



I did not struggle with sketching a graph of the given equation, $j(x) = tan(\frac{\pi}{2}x)$. I know that tangent functions have vertical wavy lines that repeat forever. I then plugged the equation into desmos, and the generated graph matched the given criteria for the equation.

pg.421, #21: If $tan(x) = -1.5$, find $tan(-x)$.

$$tan(-x) = -(-1.5)$$
$$tan(-x) = 1.5$$

I did not struggle with finding $tan(-x)$ when $tan(x)$ is given. All I had to do was make the equation given for $tan(x)$ negative and simplify it to get 1.5.

pg.421, #23: If $sec(x) = 2$, find $sec(-x)$.

$$sec(-x) = -(2)$$
$$sec(-x) = (2)$$

I found solving this type of problem very easy. All I had to do was take the given value for $sec(x)$, 2, and make it negative to get $sec(-x) = -2$.

pg.421, #27: Simplify $cot(-x)cos(-x) + sin(-x)$ completely.

$$(-cotx)(cosx) - sinx = (-\frac{cosx}{sinx})(cosx) - sinx$$

$$(-\frac{cos^2x}{sinx}) - sinx$$

$$\frac{-(cos^2x + sin^2x)}{sinx}$$

$$\frac{-1}{sinx} = -cscx$$

I did not struggle with this problem. To simplify the given equation I needed to understand the rules of sine functions and how to convert them/how they are equivalent. After understanding those rules, I just needed to follow PEMDAS and simplify to get $-cscx$.

1.3 P.429, 1-12, (Pick 2) ; P.429, 19-26, (Pick 2)

pg.429, #1: Evaluate $sin^{-1}(\frac{\sqrt{2}}{2})$.

$$sin^{-1}(\frac{\sqrt{2}}{2}) = \frac{\pi}{4}$$

I did not struggle with evaluating the expression given its answer in radians. I used the unit circle and looked for an angle in $[-\frac{\pi}{2}, \frac{\pi}{2}]$ with a sine value of $\frac{\sqrt{2}}{2}$. The angle that satisfies these conditions is $\frac{\pi}{4}$.

pg.429, #3: Evaluate $sin^{-1}(-\frac{1}{2})$.

$$sin^{-1}(-\frac{1}{2}) = -\frac{\pi}{6}$$

I found solving this type of problem very easy. I used the unit circle and was looking for an angle in $[-\frac{\pi}{2}, \frac{\pi}{2}]$ with a sine value of $(-\frac{1}{2})$. The angle that satisfies these conditions is $-\frac{\pi}{6}$.

pg.429, #19: Evaluate $sin^{-1}(cos(\frac{\pi}{4}))$.

$$sin^{-1}(cos(\frac{\pi}{4})) = sin^{-1}(\frac{\sqrt{2}}{2})$$

$$sin^{-1}(\frac{\sqrt{2}}{2}) = \frac{\pi}{4}$$

I did not struggle with this problem. After identifying that $sin^{-1}(cos(\frac{\pi}{4})) = sin^{-1}(\frac{\sqrt{2}}{2})$, I followed the same exact steps that I took in pg.429, #1. I used the unit circle and looked for an angle in $[-\frac{\pi}{2}, \frac{\pi}{2}]$ with a sine value of $\frac{\sqrt{2}}{2}$. The angle that satisfies these conditions is $\frac{\pi}{4}$.

pg.429, #21: Evaluate $sin^{-1}(cos(\frac{4\pi}{3}))$.

$$sin^{-1}(cos(\frac{4\pi}{3})) = sin^{-1}(-\frac{1}{2})$$

$$sin^{-1}(-\frac{1}{2}) = -\frac{\pi}{6}$$

I found solving this problem easy. After I identified that $sin^{-1}(cos(\frac{4\pi}{3})) = sin^{-1}(-\frac{1}{2})$, I followed the same exact steps that I took in pg.429, #3. I found solving this type of problem very easy. I used the unit circle and was looking for an angle in $[-\frac{\pi}{2}, \frac{\pi}{2}]$ with a sine value of $(-\frac{1}{2})$. The angle that satisfies these conditions is $-\frac{\pi}{6}$.

1.4 P.440, 1-8, (Pick 2) ; P.440, 9-12, (Pick 2) ; P.440, 13-24, (Pick 2) ; P.440, 33-40, (Pick 2)

pg.440, #1: Find all solutions on the interval $0 \leq \theta < 2\pi$ for $2sin(\theta) = -\sqrt{2}$.

$$sin(\theta) = \frac{-\sqrt{2}}{2}$$

$$\theta = \frac{5\pi}{4} + 2k\pi$$

$$\theta = \pi - \frac{\pi}{4} + 2k\pi = \frac{7\pi}{4} + 2k\pi$$

$$\theta = \frac{5\pi}{4}, \theta = \frac{7\pi}{4}$$

I struggled a lot with this problem. I don't really understand the math provided in the solution manual or how to start the problem. I did out the solution provided for the problem, bit I don't really understand why I did what I did to get the solution.

pg.440, #3: Find all solutions on the interval $0 \leq \theta < 2\pi$ for $2cos(\theta) = 1$.

$$cos(\theta) = \frac{1}{2}$$

$$\theta = \frac{\pi}{3} + 2k\pi$$

$$\theta = 2\pi - \frac{\pi}{3} + 2k\pi = \frac{5\pi}{3} + 2k\pi$$

$$\theta = \frac{\pi}{3}, \theta = \frac{5\pi}{3}$$

I had issues with understanding this problem as well. I completed the math provided in the solution manual but I do not really understand why I did the steps that I did and how they tie together.

pg.440, #9: Find all solutions for $2cos(\theta) = \sqrt{2}$.

$$cos(\theta) = \frac{\sqrt{2}}{2}$$

$$\theta = \frac{\pi}{4} + 2k\pi$$

$$\theta = 2\pi - \frac{\pi}{4} + 2k\pi = \frac{7\pi}{4} + 2k\pi$$

$$\theta = \frac{\pi}{4}, \theta = \frac{7\pi}{4}$$

I am having a lot of trouble with this section of the chapter. I followed the math provided in the solution manual but I do not get why I am doing the math that I am doing. I am not sure what the purpose of the addition of $2k\pi$ in all of the problems that I have completed so far.

pg.440, #11: Find all solutions for $2sin(\theta) = -1$.

$$sin(\theta) = -\frac{1}{2}$$

$$\theta = \frac{7\pi}{6} + 2k\pi$$

$$\theta = \frac{11\pi}{6} + 2k\pi$$

$$\theta = \frac{7\pi}{6}, \theta = \frac{11\pi}{6}$$

I struggled a lot with this problem. I think that I understand what it is asking, but the math provided in the solution manual is confusing me a lot. I am not sure why $2k\pi$ is added to the solutions for some reason and then removed at the end.

pg.440, #13: Find all solutions for $2sin(3\theta) = 1$.

$$sin(3\theta) = \frac{1}{2}$$
$$3\theta = \frac{\pi}{6} + 2k\pi, \theta = \frac{\pi}{18} + \frac{2k\pi}{3}$$
$$3\theta = \frac{5\pi}{6} + 2k\pi, \theta = \frac{5\pi}{18} + \frac{2k\pi}{3}$$
$$\theta = \frac{\pi}{18}, \theta = \frac{5\pi}{18}$$

I found this problem to be pretty difficult. I think that I understand what it is asking me to do, but when I follow the solution manual I get very confused. I understand the concept of having a coefficient for θ and solving with that condition, but I am still confused by the addition of $2k\pi$ to θ.

pg.440, #15: Find all solutions for $2sin(3\theta) = -\sqrt{2}$.

$$sin(3\theta) = -\frac{\sqrt{2}}{2}$$
$$3\theta = \frac{5\pi}{4} + 2k\pi, \theta = \frac{5\pi}{12} + \frac{2k\pi}{3}$$
$$3\theta = \frac{7\pi}{4} + 2k\pi, \theta = \frac{7\pi}{12} + \frac{2k\pi}{3}$$
$$\theta = \frac{5\pi}{12}, \theta = \frac{7\pi}{12}$$

I struggled with this problem a bit. I think that I understand the concept, but the solutions provided in the solution manual are very confusing. I am not sure why I am adding $2k\pi$ to θ and then ignoring it and just having θ as the answer by itself.

pg.440, #33: Find the first two positive solutions for $7sin(6x) = 2$.

$$sin(6x) = \frac{2}{7}$$
$$6x = sin^{-1}(\frac{2}{7})$$
$$6x = 0.28975 + 2k\pi$$
$$6x = \pi - 0.28975 + 2k\pi$$
$$6x = 0.28975 + 2(0)\pi, x = 0.04829$$
$$6x = \pi - 0.28975 + 2(0)\pi, x = 0.47531$$

I understood this problem more than any of the other problems in this section of the chapter. While I am still unsure of the purpose of $2k\pi$, by having k equal to 0 I knew that it would provide on the positive solutions for the given equation. After plugging in 0 for k, I simplified to get what x was equal to, the positive solutions.

pg.440, #35: Find the first two positive solutions for $5cos(3x) = -3$.

$$cos(3x) = -\frac{3}{5}$$
$$3x = cos^{-1}(-\frac{3}{5})$$
$$3x == 2.2143 + 2k\pi$$
$$3x == 2\pi - 2.2143 + 2k\pi$$
$$3x == 2.2143 + 2(0)\pi, x = 0.7381$$
$$3x == 2\pi - 2.2143 + 2(0)\pi, x = 1.3563$$

I found this problem to be semi-difficult. I used the solution manual to help me in understanding the steps I needed to take to complete the problem, but it was slightly confusing for me. I am not sure why I add $2k\pi$ to θ, but because I plugged in 0 for k to get only the positive solutions it did not matter.

1.5 P.448-451, 7-21, (Pick 4)

pg.448, #7: Outside temperature over the course of a day can be modeled as a sinusoidal function. Suppose you know the high temperature for the day is 63 degrees and the low temperature of 37 degrees occurs at 5 AM. Assuming t is the number of hours since midnight, find an equation for the temperature, D, in terms of t.

Amplitude - 13
Midline - 50
Horizontal Shift - -5

$$D(t) = -13cos(\frac{\pi}{12}(t-5)) + 50$$

I did not struggle with this problem. I used the base formula, $f(x) = asin(bx+c)+d$, and found the variables through the information provided. I found the amplitude by finding the halfway point between the maximum and minimum recorded temperatures and then diving by 2. The midline is the exact halfway point between the given maximum and minimum recorded temperatures, 50. The horizontal shift is the amount of time past since midnight, when the lowest temperature was recorded. I then plugged the variables into the formula to get $D(t) = -13cos(\frac{\pi}{12}(t-5)) + 50$.

pg.448, #9: A population of rabbits oscillates 25 above and below an average of 129 during the year, hitting the lowest value in January (t = 0).

A: Find an equation for the population, P, in terms of the months since January, t.
B: What if the lowest value of the rabbit population occurred in April instead?

a: $P(t) = -25cos(\frac{\pi}{6}(t)) + 129$
Midline - 129
Amplitude - 25

b: $P(t) = -25cos(\frac{\pi}{6}(t-3)) + 129$

I did not struggle with this word problem. To generate the equation, first I used the base formula, $f(x) = asin(bx + c) + d$, and found the variables through the given information. The midline is the halfway point between the maximum population and the minimum population. The amplitude the the halfway point between the maximum population and minimum population divided by 2. I plugged in the variables into the formula to get $P(t) = -25cos(\frac{\pi}{6}(t)) + 129$. To find the lowest value of the rabbit population occurred in April instead, all I had to do was subtract 3 months from the generated equation to get $P(t) = -25cos(\frac{\pi}{6}(t-3)) + 129$.

pg.449, #11: Outside temperature over the course of a day can be modeled as a sinusoidal function. Suppose you know the high temperature of 105 degrees occurs at 5 PM and the average temperature for the day is 85 degrees. Find the temperature, to the nearest degree, at 9 AM.

Midline - 85
Amplitude - 20
Horizontal Shift - -17

$$D(t) = 20cos(\frac{\pi}{12}(t-17)) + 85$$

$$D(9) = 20cos(\frac{\pi}{12}((9)-17)) + 85$$

$$D(9) = 75^{\circ F}$$

I did not find this problem difficult. First I needed to create a base equation for the word problem before I could solve for a certain time. To do this I used the base formula, $f(x) = asin(bx + c) + d$, and found the variables through the given information. I found the midline by finding the halfway point between the maximum temperature and the minimum temperature. The amplitude the the midline divided by 2. I plugged those variables into the formula to get $D(t) = 20cos(\frac{\pi}{12}(t-17)) + 85$. I then plugged in D(9), $D(9) = 20cos(\frac{\pi}{12}((9)-17)) + 85$, and solved to get the temperature at 9 AM, which is $75^{\circ F}$.

pg.449, #13: Outside temperature over the course of a day can be modeled as a sinusoidal function. Suppose you know the temperature varies between 47 and 63 degrees during the day and the average daily temperature first occurs at 10 AM. How many hours after midnight does the temperature first reach 51 degrees?

Midline - 55
Amplitude - 8
Horizontal Shift - -10

$$D(t) = 8sin(\frac{\pi}{12}(t-10)) + 55$$

$$51 = 8sin(\frac{\pi}{12}(t-10)) + 55$$

$$sin(\frac{\pi}{12}(t-10)) = -\frac{1}{2}$$

$$\frac{\pi}{12}(t-10) = sin^{-1}(-\frac{1}{2})$$

$$t = \frac{\frac{-\pi}{6}}{\frac{\pi}{12}} + 10 = 8$$

t = 8 AM, 8 hours past midnight

I found this problem to be very easy. I created a base equation for the word problem using the formula $f(x) = asin(bx + c) + d$. I found the variables using the information provided in the question. The midline is the halfway point between the maximum and minimum recorded temperatures. The amplitude is the midline divided by 2. The horizontal shift is how many hours have pasted since midnight, 10 hours. I then plugged the variables in to get $D(t) = 8sin(\frac{\pi}{12}(t-10)) + 55$. To find what time 51 degrees is reached at, I set D(t) equal to 51 degrees as in, $51 = 8sin(\frac{\pi}{12}(t-10)) + 55$. I then simplified the equation down until I got 8, for 8 AM or 8 hours past midnight.